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EGADS Lineage Algorithm Handbook

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Contents

Chapter 1

Introduction

This document contains descriptions of algorithms contained in the EGADS toolbox. Within each algorithm description is the following:

- **Algorithm Name** – name of algorithm as implemented in EGADS .
- **Category** – general category of algorithm. Algorithm can be found in this subdirectory in EGADS .
- **Summary** – short description of what the algorithm does.
- **Inputs** – expected inputs to algorithm. This field includes expected units, and data type of input.
- **Outputs** – outputs produced by algorithm.
- **Formula** – description of formulas or methods behind the algorithm.
- **Source** – person, institution or entity who provided the algorithm.
- **References** – any references to literature, journals or documents with more information on the current algorithm

To aid in algorithm usage and discovery, there is a general naming scheme for EGADS algorithms. Generally, algorithm names are composed as follows:
`{measurement}_{context/detail/instrument}_{source}`
For example, an algorithm provided by CNRM to calculate the density of dry air would be named `density_dry_air_cnrm`.

For more information about using these algorithms within EGADS , or using EGADS itself, please refer to the EGADS documentation which can be found at https://github.com/eufarn7sp/egads

Part I

General Algorithms

Chapter 2

Mathematics

2.1 Time Derivative

Algorithm name: derivative_wrt_time

Category: Mathematics

Summary: Calculation of the first time derivative of a generic parameter. Calculations of time derivatives are centered for all except the first and last values in the vector. Returns `None` value for scalar parameters.

Inputs:

x	Vector	Parameter to calculate first derivative
t	Vector	Time signal [sec]

Outputs:

$\dot{x}$	Vector	First derivative of x [units of x / sec]

Formula:

$$\dot{x}_i = \frac{x_{i+1} - x_{i-1}}{t_{i+1} - t_{i-1}}$$

Source:

References:

Chapter 3

Corrections

3.1 Simple correction of spikes

Algorithm name: correction_spike_simple_cnrm

Category: Corrections

Summary: Detection of spikes which exceed a specified threshold. The detected value is replaced with the mean of the surrounding values.
This algorithm does not apply well to variables that are naturally discontinuous.

Inputs:

X	Vector	Parameter for analysis
S_0	Coeff	Spike detection threshold (same units as X, and must be positive)

Outputs:

X_c	Vector	Parameter with corrections applied

Formula: The ith term is considered a spike if the following are all true:

$$\|X[i] - X[i-1]\| > S_0 \tag{3.1}$$

$$\|X[i] - X[i+1]\| > S_0 \tag{3.2}$$

$$(X[i] - X[i-1])(X[i] - X[i+1]) > 0 \tag{3.3}$$

with

$$X_c[i] = \frac{X[i+1] + X[i-1]}{2}$$

Otherwise, $X_c[i] = X[i]$

Source: CNRM/GMEI/TRAMM

References:

Chapter 4

Transforms

4.1 Linear Interpolation

Algorithm name: interpolate_linear

Category: Transforms

Summary: This algorithm linearly interpolates a variable piecewise from one coordinate system to another. It is mostly used to fill gaps.

Inputs:

x	Vector	x-coordinates of the data points (must be increasing).
f	Vector	Data points to interpolate.
x_{interp}	Vector	New set of x-coordinates to use in interpolation.
f_{left}	Coeff, optional	Value to return when $x_{interp} < x_0$. Default is f_0.
f_{right}	Coeff, optional	Value to return when $x_{interp} > x_n$. Default is f_n.

Outputs:

f_{interp}	Vector	Interpolated values of f.

Formula: For each value of x_{interp} the two surrounding points are found and designated x_a and x_b, with corresponding values f_a and f_b. Then f_{interp} is calculated piecewise as follows:

$$f_{interp}[i] = f_a + (x_{interp}[i] - x_a)\frac{f_b - f_a}{x_b - x_a}$$

Values where x_{interp} is less than x_0 are replaced with f_{left}, if provided, or f_0. Likewise, f_{right} if given, or f_n are substituted where x_{interp} is greater than x_n.

Important: in the current version of the algorithm, the corresponding i^{th} value is interpolated only if:

- $x_{interp}[i]$ doesn't exist in x
- $f(x) = NaN$ if $x_{interp}[i]$ exists in x

Source:

References:

4.2 Linear Interpolation (old)

Algorithm name: interpolate_linear_old

Category: Transforms

Summary: This algorithm linearly interpolates a variable piecewise from one coordinate system to another. All values are interpolated, even if they exist in the new coordinate system.

Inputs:

x	Vector	x-coordinates of the data points (must be increasing).
f	Vector	Data points to interpolate.
x_{interp}	Vector	New set of x-coordinates to use in interpolation.
f_{left}	Coeff, optional	Value to return when $x_{interp} < x_0$. Default is f_0.
f_{right}	Coeff, optional	Value to return when $x_{interp} > x_n$. Default is f_n.

Outputs:

f_{interp}	Vector	Interpolated values of f.

Formula: For each value of x_{interp} the two surrounding points are found and designated x_a and x_b, with corresponding values f_a and f_b. Then f_{interp} is calculated piecewise as follows:

$$f_{interp}[i] = f_a + (x_{interp}[i] - x_a)\frac{f_b - f_a}{x_b - x_a}$$

Values where x_{interp} is less than x_0 are replaced with f_{left}, if provided, or f_0. Likewise, f_{right} if given, or f_n are substituted where x_{interp} is greater than x_n.

Source:

References:

4.3 Convert ISO 8601 time to date/time elements

Algorithm name: isotime_to_elements

Category: Transforms

Summary: This algorithm takes a series of ISO 8601 strings and splits them into their composant values (year, month, day, hour, minute, second) using the Python dateutil module. This module is format agnostic, and will recognize any ISO 8601 format.

Inputs:

t_{ISO}	Vector	ISO 8601 date-time string

Outputs:

year	Vector	year
month	Vector	month
day	Vector	day
hour	Vector	hour
minute	Vector	minute
second	Vector	second

Formula: This algorithm applies the Python dateutil.parser module to decompose an ISO date-time string into its composant values.

Source:

References:

4.4 Convert ISO 8601 time string to seconds

Algorithm name: isotime_to_seconds

Category: Transforms

Summary: This algorithm converts a series of ISO 8601 date-time strings to delta time in seconds. It takes an optional format string for the conversion and an optional reference time. If no reference time is provided, then Jan 1, 1970, 00:00:00 is used as the reference.

Inputs:

t_{ISO}	Vector	ISO 8601 strings
t_{ISOref}	String, Optional	Reference time [ISO 8601 string] - default is '19700101T000000'
$format$	String, Optional	ISO 8601 string format - if none provided, alg will attempt to deconstruct time string.

Outputs:

Δt	Vector	Seconds since reference

Formula: This algorithm uses the Python dateutil and datetime modules to parse and process ISO 8601 date strings into seconds elapsed. The basic steps of the algorithms are:

1. Convert from ISO 8601 string into datetime tuple. If no format string is used, the Python function dateutil.parser.parse is used to deconstruct the string, since it can automatically recognize nearly any date string format. If a format string is provided, then datetime.datetime.strptime(string, format) is used to deconstruct the string.

2. datetime tuple objects are subtracted from the reference time to get a datetime.timedelta object.

3. Number of seconds and microseconds are calculated from the datetime.timedelta object and stored as numeric objects and passed out of the algorithm.

Source:

References:

4.5 Convert elapsed seconds to ISO 8601 time string

Algorithm name: seconds_to_isotime

Category: Transforms

Summary: Given a vector of elapsed seconds and a reference time, this algorithm calculates a series of ISO 8601 formatted time strings using the Python datetime module. The format of the returned ISO 8601 strings can be controlled by the optional $format$ parameter. The default format is yyyymmddTHH-MMss.

Inputs:

t_{secs}	Vector	Elapsed seconds [s]
t_{ref}	String	ISO 8601 reference time
$format$	String, optional	ISO 8601 format string, default is yyyymmddTHH-MMss

Outputs:

t_{ISO}	Vector	ISO 8601 date-time strings

Formula: The ISO 8601 time strings are generated from the inputs using the Python datetime module using these steps for each item in the t_{secs} vector:

1. Create a datetime object using the input reference time (t_{ref}) representing the start time.
2. Calculate a timedelta object from the input elapsed seconds parameter.
3. Add the timedelta object to the reference datetime object to calculate an absolute time.
4. Convert the resulting datetime object to an ISO 8601 string following the given $format$, if any.

Source:

References:

4.6 Converts a time or a time vector to decimal year.

Algorithm name: time_to_decimal_year

Category: Transforms

Summary: Given a vector of time ($ms/s/mm/h/d/m$) and an optional reference year, this algorithm converts the data to a format in decimal year. Ex: 1995.0125

Inputs:

t	Vector	Time [s]
t_{ref}	String, optional	Time reference, default is 19500101T000000

Outputs:

t_y	Vector	Time in decimal year [year]

Formula: The decimal year vector t_y is generated from the inputs using the Python datetime module using these steps for each item in the t vector:

1. Regardless of the time format (second, minute, hour, day, month, ...), t is converted to year automatically by the instance EgadsData.
2. The user time reference, t_{ref}, if provided by the user, is converted to seconds using the algorithm ISOtimeToSeconds, based on the reference 1950-01-01 at 00h00mm00s. t_{ref} can be positive if the user time reference is after 1950-01-01, or negative if the user time reference is before 1950-01-01.
3. The time reference is then rescaled to year.
4. The final t_y vector is computed by adding t_{ref} + 1950 to t.

Source:

References:

Part II

Atmospheric Algorithms

Chapter 5

Thermodynamics

5.1 Incremantal pressure altitude

Algorithm name: altitude_pressure_incremental_cnrm

Category: Thermodynamics

Summary: Calculate a pressure altitude incrementally along the trajectory of an aircraft from the Laplace formula (Z2 = Z1 + Ra/g < Tv > log(P1/P2)).

Inputs:

Ps	Vector[t]	Static pressure [hPa]
Tv	Vector[t]	Virtual temperature [K or °C]
t	Vector[t]	Measurement period [s]
$Z0$	Coeff	Reference altitude at S0 if S0 is provided, can be airport altitude (m) if S0 is not provided and measurements start in airport [m]
$S0$	Coeff, optional	Reference time, if not provided S0 = t[0] [s]

Outputs:

alt_p	Vector[t]	Pressure altitude [m]

Formula: Tv is converted to Kelvin if needed, then:

$$Z_{i0} = Z0 \text{ with } i0 \text{ such as } ref\_time_{i0} = S0$$

$$Zj = Zi + \frac{R_a}{g} \cdot \left(\frac{Tv_j + Tv_i}{2}\right) \cdot \log\left(\frac{Ps_i}{Ps_j}\right) \text{ with } \begin{cases} i = j + 1 \text{ for } j < i0 \\ i = j - 1 \text{ for } j > i0 \end{cases}$$

Source: CNRM/GMEI/TRAMM

References: Equation of state for a perfect gas, Triplet-Roche [10], page 36.

5.2 Pressure altitude

Algorithm name: altitude_pressure_raf

Category: Thermodynamics

Summary: Calculates pressure altitude given static pressure using US Standard Atmosphere definitions. Sea level conditions in the US Standard Atmosphere are defined as having a pressure of 1013.25 hPa and a temperature of 15 degC at an altitude of 0m.

Inputs:

P_s	Vector	Static pressure [hPa]

Outputs:

H	Vector	Pressure altitude [m]

Formula: For pressures greater than or equal to 226.3206:

$$H = \frac{T_0}{L}\left[1 - \left(\frac{P_s}{P_0}\right)^{\frac{R_a L}{g}}\right]$$

where the lapse rate L is 0.0065 K/m. For pressures less than 226.3206:

$$H = H_1 + \frac{R_a T_1}{g}\ln\left(\frac{P_1}{P_s}\right)$$

where H_1 is 11000m, T_1 is 216.65 K and P_1 is 226.3206.

Source: NCAR EOL-RAF

References: US Standard Atmosphere 1976 (NASA-TM-X-74335), 241 pages. http://ntrs.nasa.gov/archiv

5.3 Density of dry air

Algorithm name: density_dry_air_cnrm

Category: Thermodynamics

Summary: Calculates density of dry air given static temperature and pressure.

Inputs:

P_s	Vector	Static pressure [hPa]
T_s	Vector	Static temperature [K or °C]

Outputs:

ρ	Vector	Density of dry air [kg/m^3]

Formula:

$$\rho = \frac{100P_s}{R_a T_s}$$

with $R_a = 287.05$ J kg^{-1} K^{-1}

Density of humid air can be calculated using this same algorithm by using virtual temperature instead of static temperature.

Source: CNRM/GMEI/TRAMM

References: Equation of state for a perfect gas, Triplet-Roche [10], page 34.

5.4 Relative humidity from capacitive probe

Algorithm name: hum_rel_capacitive_cnrm

Category: Thermodynamics

Summary: Calculates relative humidity using the measured frequency from a capacitive probe.

Inputs:

$Ucapf$	Vector	Output frequency of the capacitive probe [Hz]
T_s	Vector	Static temperature [K]
P_s	Vector	Static pressure [hPa]
ΔP	Vector	Dynamic pressure [hPa]
C_t	Coeff.	Temperature correction coefficient [%°C]
F_{min}	Coeff.	Minimal acceptable frequency [Hz]
C_0	Coeff.	0th degree calibration coefficient
C_1	Coeff.	1st degree calibration coefficient
C_2	Coeff.	2nd degree calibration coefficient

Outputs:

H_u	Vector	Relative humidity [%]

Formula: If $Ucapf \leq F_{min}$ then $Ucapf = F_{min}$

$$H_u = \frac{P_s}{P_s + \Delta P}\left[C_0 + C_1 Ucapf + C_2 Ucapf^2 + C_t(T_s - 20)\right]$$

with T_s in $°C$ and 20 in $°C$.

Source: CNRM/GMEI/TRAMM

References: CAM note on humidity instrument measurements. [1]

5.5 Pressure and angle of incidence (CNRM)

Algorithm name: pressure_angle_incidence_cnrm

Category: Thermodynamics

Summary: Calculates static pressure and dynamic pressure by correction of static error. Angle of attack and sideslip are calculated from the horizontal and vertical differential pressures.

Inputs:

P_{sr}	Vector	Raw static pressure [hPa]
ΔP_r	Vector	Raw dynamic pressure [hPa]
ΔP_h	Vector	Horizontal differential pressure [hPa]
ΔP_v	Vector	Vertical differential pressure [hPa]
C_α	Coeff.[2]	Angle of attack calibration coefficients
C_β	Coeff.[2]	Slip calibration coefficients
$C_{errstat}$	Coeff.[4]	Static error coefficients

Outputs:

P_s	Vector	Static Pressure [hPa]
ΔP	Vector	Dynamic pressure corrected with static error [hPa]
α	Vector	Angle of attack [rad]
β	Vector	Sideslip [rad]

Formula: If $\Delta P_r > 25$hPa:

$$Errstat = C_{errstat}[0] + C_{errstat}[1]\Delta P_r + C_{errstat}[2]\Delta P_r^2 + C_{errstat}[3]\Delta P_r^3$$

otherwise:

$$\begin{aligned} Errstat &= \frac{\Delta P_r}{25}\text{ Errstat @ 25 hPa} \\ P_s &= P_{sr} - Errstat \\ \Delta P &= \Delta P_r + Errstat \\ \alpha &= C_\alpha[0] + C_\alpha[1]\frac{\Delta P_v}{\Delta P} \\ \beta &= C_\beta[0] + C_\beta[1]\frac{\Delta P_h}{\Delta P} \end{aligned} \tag{5.1}$$

Source: CNRM/GMEI/TRAMM

References:

5.6 Dynamic pressure and angle of incidence

Algorithm name: pressure_dynamic_angle_incidence_vdk

Category: Thermodynamics

Summary: This algorithm calculates dynamic pressure and angles of incidence from a 5-hole probe using differences in pressure between the ports. The algorithm requires calibration coefficients which are obtained by a calibration procedure of the probe at predefined airflow angles. See van den Kroonenberg, 2008 [11] for more details on the calibration procedure.

Inputs:

ΔP_t	Vector	Pressure difference between top port and center port [hPa]
ΔP_b	Vector	Pressure difference between bottom port and center port [hPa]
ΔP_l	Vector	Pressure difference between left port and center port [hPa]
ΔP_r	Vector	Pressure difference between right port and center port [hPa]
ΔP_{0s}	Vector	Pressure difference between center port and static pressure [hPa]
a_{ij}	Coeff[11,11]	Angle of attack calibration coefficients
b_{ij}	Coeff[11,11]	Sideslip calibration coefficients
q_{ij}	Coeff[11,11]	Dynamic pressure calibration coefficients

Outputs:

q	Vector	Dynamic pressure [hPa]
α	Vector	Angle of attack [deg]
β	Vector	Sideslip angle [deg]

Formula: Total pressure difference is calculated using pressure differentials from the 5 ports.

$$\Delta P = \left(\frac{1}{125}[(\Delta P_t + \Delta P_r + \Delta P_b + \Delta P_l)^2 + (-4\Delta P_t + \Delta P_r + \Delta P_b + \Delta P_l)^2 + (\Delta P_t - 4\Delta P_r + \Delta P_b + \Delta P_l)^2 + (\Delta P_t + \Delta P_r - 4\Delta P_b + \Delta P_l)^2 + (\Delta P_t + \Delta P_r + \Delta P_b - 4\Delta P_l)^2]\right)^{1/2} + \frac{1}{4}(\Delta P_t + \Delta P_r + \Delta P_b + \Delta P_l)$$

The dimensionless pressure coefficients k_α and k_β are defined using ΔP and the measured differential pressures.

$$k_\alpha = \frac{\Delta P_t - \Delta P_b}{\Delta P}$$
$$k_\beta = \frac{\Delta P_r - \Delta P_l}{\Delta P}$$

These are applied to general calibration polynomial form (11th order) from Bohn and Simon, 1975 [3], where $m = n = 11$.

$$\tilde{\alpha} = \sum_{i=0}^{m} (k_\alpha)^i \left[\sum_{j=0}^{n} a_{ij} (k_\beta)^j \right]$$
$$\tilde{\beta} = \sum_{i=0}^{m} (k_\alpha)^i \left[\sum_{j=0}^{n} b_{ij} (k_\beta)^j \right]$$
$$k_q = \sum_{i=0}^{m} (k_\alpha)^i \left[\sum_{j=0}^{n} q_{ij} (k_\beta)^j \right]$$

Finally, the dynamic pressure, angle of attack and sideslip angle can be calculated using these coefficients.

$$q = \Delta P_{0s} + \Delta P k_q$$
$$\alpha = \tilde{\alpha}$$
$$\beta = \arctan\left(\frac{\tan \tilde{\beta}}{\cos \tilde{\alpha}}\right)$$

Source:

References:

A.C. van der Kroonenberg, et al., “Measuring the Wind Vector Using the Autonomous Mini Aerial Vehicle M^{2}AV,” *J. Atmos. Oceanic Technol.*, 25 (2008): 1969-1982. [11]

5.7 Potential Temperature

Algorithm name: temp_potential_cnrm

Category: Thermodynamics

Summary: Calculates potential temperature.

Inputs:

T_s	Vector	Static temperature [K or ◦C]
P_s	Vector	Static pressure [hPa]
R_a/c_{pa}	Coeff.	Gas constant of air divided by specific heat of air at constant pressure

Outputs:

θ	Vector	Potential temperature [same unit as T_s]

Formula:

$$\theta = T_s \left(\frac{1000}{P_s}\right)^{R_a/c_{pa}}$$

Source: CNRM/GMEI/TRAMM

References: Triplet-Roche [10].

5.8 Static Temperature

Algorithm name: temp_static_cnrm

Category: Thermodynamics

Summary: Calculates static temperature of the air from total temperature. This method applies to probe types such as the Rosemount.

Inputs:

T_t	Vector	Measured total temperature [K]
ΔP	Vector	Dynamic pressure [hPa]
P_s	Vector	Static pressure [hPa]
r_f	Coeff.	Probe recovery coefficient
R_a/c_{pa}	Coeff.	Gas constant of air divided by specific heat of air at constant pressure

Outputs:

T_s	Vector	Static temperature [K]

Formula:

$$T_s = \frac{T_t}{1 + r_f\left(\left(1 + \frac{\Delta P}{P_s}\right)^{R_a/c_{pa}} - 1\right)}$$

Source: CNRM/GMEI/TRAMM

References:

5.9 Virtual Temperature

Algorithm name: temp_virtual_cnrm

Category: Thermodynamics

Summary: Calculates the virtual temperature of air.

Inputs:

T_s	Vector	Static temperature [K or ∘C]
r	Vector	Water vapor mixing ratio [g/kg]

Outputs:

T_v	Vector	Virtual temperature [same units as T_s]

Formula:

$$T_v = T_s \frac{1 + (R_v/R_a)r}{1 + r}$$

where $R_v/R_a = 1.608$

Source: CNRM/GMEI/TRAMM

References: Triplet-Roche [10], page 56.

5.10 Mach number

Algorithm name: velocity_mach_raf

Category: Thermodynamics

Summary: Calculates the mach number based on dynamic and static pressure.

Inputs:

ΔP	Vector	Dynamic pressure [hPa]
P_s	Vector	Static pressure [hPa]

Outputs:

M	Vector	Mach number

Formula:

$$M = \sqrt{\frac{2}{\gamma - 1}\left[\left(\frac{\Delta P}{P_s} + 1\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}$$

Source: NCAR-EOL

References: NCAR-RAF Bulletin #23 [7]

5.11 True air speed (CNRM)

Algorithm name: velocity_tas_cnrm

Category: Thermodynamics

Summary: Calculates true air speed based on static pressure, static temperature and dynamic pressure using the Barré-St Venant formula.

Inputs:

T_s	Vector	Static temperature [K]
ΔP	Vector	Dynamic pressure [hPa]
P_s	Vector	Static pressure [hPa]
c_{pa}	Coeff.	Specific heat of air at constant pressure (for dry air 1004 J K^{-1} kg^{-1})
R_a/c_{pa}	Coeff.	Gas constant of air divided by specific heat of air at constant pressure

Outputs:

V_t	Vector	True air speed [m/s]

Formula:

$$V_t = \sqrt{2c_{pa}T_s\left[\left(1+\frac{\Delta P}{P_s}\right)^{R_a/c_{pa}}-1\right]}$$

Source: CNRM/GMEI/TRAMM

References: NCAR-RAF Bulletin #23 [7], *Méchanique des fluides*, Candel [4]

5.12 True air speed (RAF)

Algorithm name: velocity_tas_raf

Category: Thermodynamics

Summary: Calculates true air speed based on Mach number, measured temperature and thermometer recovery factor. Typical values of the themometer recovery factor range from 0.75-0.9 for platinum wire ratiometer (flush bulb type) thermometers, and around 1.0 for TAT type thermometers.

Inputs:

T_r	Vector	Measured temperature [K]
M	Vector	Mach number
e	Coeff.	thermometer recovery factor

Outputs:

V_t	Vector	True air speed [m/s]

Formula:

$$V_t = \sqrt{\frac{R\gamma T_r M^2}{1 + 0.5(\gamma - 1)eM^2}}$$

where the recovery factor e can be determined for a thermometer by comparing its measured temperature with the actual total and static temperature.

$$e \equiv \frac{T_r - T_s}{T_t - T_s}$$

Source: NCAR-EOL

References: NCAR-RAF Bulletin #23 [7]

5.13 Longitudinal true airspeed

Algorithm name: velocity_tas_longitudinal_cnrm

Category: Thermodynamics

Summary: Calculates the true air speed along the longitudinal axis of the aircraft.

Inputs:

V_t	Vector	True air speed [m/s]
α	Vector	Angle of attack [rad]
β	Vector	Sideslip angle [rad]

Outputs:

V_{tx}	Vector	Longitudinal true air speed [m/s]

Formula:

$$V_{tx} = \frac{V_t}{\sqrt{1 + \tan^2 \alpha + \tan^2 \beta}}$$

Source: CNRM/GMEI/TRAMM

References: NCAR-RAF Bulletin #23 [7]

5.14 3D Wind Vectors

Algorithm name: wind_vector_3d_raf

Category: Thermodynamics

Summary: This algorithm applies vector transformations using aircraft speed, angle of attack and sideslip to calculate the three-dimensional wind vector components.

Inputs:

U_a	Vector	Corrected true air speed [m/s]
α	Vector	Aircraft angle of attack [rad]
β	Vector	Aircraft sideslip [rad]
u_p	Vector	Easterly aircraft velocity from INS [m/s]
v_p	Vector	Northerly aircraft velocity from INS [m/s]
w_p	Vector	Upward aircraft velocity from INS [m/s]
ϕ	Vector	Roll [rad]
θ	Vector	Pitch [rad]
ψ	Vector	True Heading [rad]
$\dot{\theta}$	Vector	Pitch rate [rad/sec]
$\dot{\psi}$	Vector	Yaw rate [rad/sec]
L	Vector	Distance separating INS and gust probe along aircraft center line [m]

Outputs:

u	Vector	Easterly wind velocity component [m/s]
v	Vector	Northerly wind velocity component [m/s]
w	Vector	Upwards wind velocity component (positive up) [m/s]

Formula:

$$D = \sqrt{(1 + \tan^2\alpha + \tan^2\beta)}$$

$$\begin{aligned}
u = &-U_a D^{-1}\left[\sin\psi\cos\theta + \tan\beta(\cos\psi\cos\phi + \sin\psi\sin\theta\sin\phi) + \tan\alpha(\sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi)\right] \\
&+ u_p - L(\dot{\theta}\sin\theta\sin\psi - \dot{\psi}\cos\psi\cos\theta) \\
v = &-U_a D^{-1}\left[\cos\psi\cos\theta - \tan\beta(\sin\psi\cos\phi - \cos\psi\sin\theta\sin\phi) + \tan\alpha(\cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi)\right] \\
&+ v_p - L(\dot{\psi}\sin\psi\cos\theta + \dot{\theta}\cos\psi\sin\theta) \\
w = &-U_a D^{-1}(\sin\theta - \tan\beta\cos\theta\sin\phi - \tan\alpha\cos\theta\cos\phi) + w_p + L\dot{\theta}\cos\theta
\end{aligned}$$

Source:

References: NCAR-RAF Bulletin #23 [7]

Chapter 6

Microphysics

6.1 Effective diameter

Algorithm name: diameter_effective_dmt

Category: Microphysics

Summary: Calculates effective diameter of a size distribution. In general, this definition is only meaningful for water clouds, and another form must be used when in ice clouds.

Inputs:

c_i	Array[time, bins]	Number concentration of hydrometeors in size category i [cm^{-3}]
d_i	Vector[bins]	Average diameter in size category i [μm]

Outputs:

D_e	Vector[time]	Effective diameter [μm]

Formula:

$$D_e = \frac{3\sum_{i=1}^{m} c_i d_i^3}{4\sum_{i=1}^{m} c_i d_i^2}$$

Source:

References: "Data Analysis User's Guide Chapter I: Single Particle Light Scattering," Droplet Measurement Technologies, 30. [5]

6.2 Mean diameter

Algorithm name: diameter_mean_raf

Category: Microphysics

Summary: Calculates the arithmetic average of all particle diameters given in a particle size distribution.

Inputs:

n_i	Array[time, bins]	Number of particles in each channel i
d_i	Vector[bins]	Channel i size [μm]

Outputs:

$\bar{D}$	Vector[time]	Mean diameter [μm]

Formula:

$$\bar{D} = \frac{\sum_i n_i d_i}{N_t}$$

where N_t is the total number of particles.

Source: NCAR-RAF

References: NCAR-RAF Bulletin No. 24. [8]

6.3 Median Volume Diameter

Algorithm name: diameter_median_volume_dmt

Category: Microphysics

Summary: Calculates the median volume diameter given a size distribution. The median volume diameter is the size of droplet below which 50% of the total water volume resides.

Inputs:

c_i	Array[time, bins]	Number concentration of hydrometeors in size category i [cm^{-3}]
d_i	Vector[bins]	Average diameter of size category i [μm]
s_i	Array[time, bins],Optional	Shape factor of the hydrometeor of size category i to account for asphericity
ρ_i	Vector[bins],Optional	Density of hydrometeor in size category i [g cm^{-3}]. Default is ρ_w = 1.0 g cm$^-3$

Outputs:

D_{mvd}	Vector[time]	Median volume diameter [μm]

Formula: Step 1: Compute liquid water content

$$W = \frac{\pi}{6}\sum_{i=1}^{m} c_i d_i^3 \rho_i s_i$$

Step 2: Beginning at the first size channel, calculate the accumulated mass $S_n = w_1 + w_2 + ...w_n$ where w_1 is the mass of water in channel 1, and w_n is the channel where the accumulated mass is greater than or equal to $0.5W$, i.e. greater than or equal to 50% of the total LWC.

Step 3: Compute the median volume diameter, D_{mvd} by interpolating linearly between the channels that bracket where the accumulated mass exceeded the total LWC:

$$D_{mvd} = d_{n-1} + (0.5 - S_{n-1}/S_n)(d_n - d_{n-1})$$

Source:

References: "Data Analysis User's Guide Chapter I: Single Particle Light Scattering,“ Droplet Measurement Technologies, 33. [5]

6.4 Extinction Coefficient

Algorithm name: extinction_coeff_dmt

Category: Microphysics

Summary: Calculates extinction coefficient given a particle size distribution.

Inputs:

c_i	Array[time, bins]	Number concentration of hydrometeors in size category i [cm^{-3}]
d_i	Vector[bins]	Average diameter of size category i [μm]
Q_e	Vector[bins], Optional	Extinction efficiency; default is $Q_e = 2$

Outputs:

B_e	Vector[time]	Extinction coefficient [km^{-1}]

Formula:

$$B_e = \frac{\pi}{4} \sum_{i=1}^{m} Q_e c_i d_i^2$$

Source:

References: "Data Analysis User's Guide Chapter I: Single Particle Light Scattering," Droplet Measurement Technologies, 30. [5]

6.5 Mass Concentration

Algorithm name: mass_conc_dmt

Category: Microphysics

Summary: Calculates mass concentration given a size distribution. Can be used to calculate liquid or ice water content depending on the types of hydrometeors being sampled.

Inputs:

c_i	Array[time, bins]	Number concentration of hydrometeors in size category i [cm^{-3}]
d_i	Vector[bins]	Average diameter of size category i [μm]
s_i	Array[time, bins]	Shape factor of the hydrometeor of size category i to account for asphericity
ρ_i	Vector[time, bins]	Density of the hydrometeor in size category i [g cm^{-3}]

Outputs:

M	Vector[time]	Mass concentration [g cm^{-3}]

Formula:

$$M = \frac{\pi}{6}\sum_{i=1}^{m} s_i \rho_i c_i d_i^3$$

Source:

References: “Data Analysis User’s Guide Chapter I: Single Particle Light Scattering,“ Droplet Measurement Technologies, 30. [5]

6.6 Total Number Concentration (DMT)

Algorithm name: number_conc_total_dmt

Category: Microphysics

Summary: Calculation of total number concentration given distribution of particle counts from a particle sampling probe.

Inputs:

c_i	Array[time, bins]	Number concentration of hydrometeors in size category i [cm^{-3}]

Outputs:

N	Vector[time]	Total number concentration [cm^{-3}]

Formula:

$$N = \sum_{i=1}^{m} c_i$$

Source:

References: "Data Analysis User's Guide Chapter I: Single Particle Light Scattering," Droplet Measurement Technologies, 30. [5]

6.7 Total Number Concentration

Algorithm name: number_conc_total_raf

Category: Microphysics

Summary: Calculation of total number concentration for a particle probe.

Inputs:

n_i	Array	Number of particles in each channel i
SV	Array	Sample volume [m^3]

Outputs:

N_t	Vector	Total number concentration [m^{-3}]

Formula:

$$N_t = \sum_i \frac{n_i}{SV_i}$$

Source: NCAR-RAF

References: NCAR-RAF Bulletin No. 24. [8]

6.8 Sample area for imaging probes (All in)

Algorithm name: sample_area_oap_all_in_raf

Category: Microphysics

Summary: Calculation of 'all in' sample area size for OAP probes such as the 2DC, 2DP, CIP, etc. This sample area varies by number of shadowed diodes. This routine calculates a sample area per bin.

Inputs:

λ	Coeff.	Laser wavelength [nm]
D_{arms}	Coeff.	Distance between probe arm tips [mm]
dD	Coeff.	Diode diameter [μm]
M	Coeff.	Probe magnification factor
N	Coeff.	Number of diodes in array

Outputs:

SA	Vector	Sample area [m^2]

Formula:

$$DOF_i = \frac{6R_i^2}{\lambda} \tag{6.1}$$
$$R_i = i\frac{dD}{2}$$
$$X = 1...N-1$$

where DOF must be less than D_{arms}. The parameter i ranges from 1 to $N-1$, since particles touching either edge are rejected as they are not considered 'all-in'.

$$ESW_i = \frac{dD(N - X_i - 1)}{M}$$

A value for ESW_i (effective sample width) is calculated for each X.

$$SA_i = (DOF_i)(ESW_i)$$

Source: NCAR-RAF

References: NCAR-RAF Bulletin No. 24. [8]

6.9 Sample area for imaging probes (Center In)

Algorithm name: sample_area_oap_center_in_raf

Category: Microphysics

Summary: Calculation of 'center in' sample area size for OAP probes such as the 2DC, 2DP, CIP, etc. This sample area varies by number of shadowed diodes. This routine is intended to calculate a sample area per bin.

Inputs:

λ	Coeff.	Laser wavelength [nm]
D_{arms}	Coeff.	Distance between probe arm tips [mm]
dD	Coeff.	Diode diameter [μm]
M	Coeff.	Probe magnification factor
N	Coeff.	Number of diodes in array

Outputs:

SA	Vector	Sample area [m^2]

Formula:

$$DOF_i = \frac{6R_i^2}{\lambda} \tag{6.2}$$
$$R_i = X\frac{dD}{2}$$
$$X = 1...N$$

where DOF must be less than D_{arms}. The parameter i ranges from 1 to N.

$$ESW = \frac{NdD}{M}$$

$$SA_i = (DOF_i)(ESW)$$

Source: NCAR-RAF

References: NCAR-RAF Bulletin No. 24. [8]

6.10 Sample area for scattering probes

Algorithm name: sample_area_scattering_raf

Category: Microphysics

Summary: Calculation of sample area for scattering probes such as the FSSP, CAS, etc.

Inputs:

DOF	Coeff.	Depth of field [m]
BD	Coeff.	Beam diameter [m]

Outputs:

SA	Coeff.	Sample area [m^2]

Formula:

$$SA = (DOF)(BD)$$

Source: NCAR-RAF

References: NCAR-RAF Bulletin No. 24. [8]

6.11 Sample Volume

Algorithm name: sample_volume_general_raf

Category: Microphysics

Summary: Calculates sample volume for microphysics probes (1D, 2D, FSSP, etc).

Inputs:

V_t	Vector	True air speed [m/s]
SA	Coeff.	Sample area of probe [m^2]
t_s	Coeff.	Sample rate [s]

Outputs:

SV	Vector	Sample volume [m^3]

Formula:

$$SV = V_t t_s SA$$

Source: NCAR-RAF

References: NCAR-RAF Bulletin No. 24. [8]

6.12 Surface Area Concentration

Algorithm name: surface_area_conc_dmt

Category: Microphysics

Summary: Calculation of surface area concentration given size distribution from particle probe.

Inputs:

c_i	Array[time, bins]	Number concentration of hydrometeors in size category i [cm^{-3}]
d_i	Vector[bins]	Average diameter of size category i [μm]
s_i	Array[time, bins]	Shape factor of hydrometeor in size category i, to account for asphericity

Outputs:

S	Vector[time]	Surface area concentration [μm^2 cm^{-3}]

Formula:

$$S = \pi \sum_{i=1}^{m} s_i c_i d_i^2$$

Source:

References: "Data Analysis User's Guide Chapter I: Single Particle Light Scattering," Droplet Measurement Technologies, 30. [5]

Chapter 7

Radiation

7.1 Camera Viewing Angles

Algorithm name: camera_viewing_angles

Category: Radiation

Summary: Calculates per-pixel camera viewing angles of a digital camera given its sensor dimension and focal length. x–y coordinates are defined as having the left side of the image (x=0) aligned with the flight direction and y=0 to the top of the image.

Inputs:

n_x	Coeff	Number of pixels in x direction
n_y	Coeff	Number of pixels in y direction
l_x	Coeff	Length of the camera sensor in x direction [mm]
l_y	Coeff	Length of the cameras sensor in y direction [mm]
f	Coeff	Focal length of the camera lens [mm]

Outputs:

θ_c	Array[n_x,n_y]	Camera viewing zenith angle [deg]
Φ_c	Array[n_x,n_y]	Camera viewing azimuth angle [deg], mathematic negative system with 0°into flight direction, clockwise

Formula:

For each i, j where $0 < i < n_x$ and $0 < j < n_y$:

$$x = l_x \frac{(i - n_x/2)}{n_x}$$
$$y = l_y \frac{(i - n_y/2)}{n_y}$$
$$d = \sqrt{x^2 + y^2}$$

$$\theta_c(i,j) = 2\tan^{-1}\frac{d}{2f}$$
$$\Phi_c(i,j) = 2\pi - \tan^{-1}\frac{y}{x}$$

Source: Andre Ehrlich, Leipzig Institute for Meteorology (a.ehrlich@uni-leipzig.de)

References:

7.2 Planck Emission

Algorithm name: planck_emission

Category: Radiation

Summary: Calculates the Planck emission of a surface at a given wavelength given its temperature.

Inputs:

T	Vector	Temperature [K]
λ	Coeff	Wavelength [nm]

Outputs:

rad	Vector	Black body radiance [W m-2 sr-1 nm-1]

Formula: After converting λ to meters, the radiance is calculated by:

$$rad = \frac{2hc^2}{\lambda^5(\exp(\frac{hc}{k_B\lambda T}) - 1)} * 10^{-9}$$

where c is the speed of light in m/s, h is the Planck constant in J s and k_B is the Boltzmann constant in J/K.

Source: Andre Ehrlich, Leipzig Institute for Meteorology (a.ehrlich@uni-leipzig.de)

References:

7.3 Rotate solar vector to aircraft frame

Algorithm name: rotate_solar_vector_to_aircraft_frame

Category: Radiation

Summary: Rotates solar vector to aircraft coordinates given roll, pitch and yaw. All rotations are defined with a mathematically positive spherical coordinate system.

Inputs:

$\theta_\odot$	Vector	Solar Zenith [degrees]
$\Phi_\odot$	Vector	Solar Azimuth [degrees] (mathematic negative, North=0°, clockwise)
ϕ_a	Vector	Aircraft roll angle [degrees] (mathematic positive, left wing up=positive)
θ_a	Vector	Aircraft pitch angle [degrees] (mathematic positive, nose down=positive)
ψ_a	Vector	Aircraft yaw angle [degrees] (mathematic negative, North=0°, clockwise)

Outputs:

$\theta_{\odot a}$	Vector	Solar Zenith, AC coordinates [degrees]
$\Phi_{\odot a}$	Vector	Solar Azimuth, AC coordinates [degrees] (mathematic negative, North=0°, clockwise)

Formula: First, $\Phi_\odot$ and ψ_a must be transformed into mathematially positive coordinate systems by subtracting them from 360.

Next, the cartesian coordinates are calculated from the solar vector:

$$x = \sin\theta_\odot \cos\Phi_\odot$$
$$y = \sin\theta_\odot \sin\Phi_\odot$$
$$z = \cos\theta_\odot$$

Then, the cartesian coordinates are rotated using three rotation matrixes using yaw, pitch and roll:

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos\theta_a \cos\psi_a & \cos\theta_a \sin\psi_a & -\sin\theta_a \\ \sin\phi_a \sin\theta_a \cos\psi_a - \cos\phi_a \sin\psi_a & \sin\phi_a \sin\theta_a \sin\psi_a + \cos\phi_a \cos\psi_a & \sin\phi_a \cos\theta_a \\ \cos\phi_a \sin\theta_a \cos\psi_a + \sin\phi_a \sin\psi_a & \cos\phi_a \sin\theta_a \sin\psi_a - \sin\phi_a \cos\psi_a & \cos\phi_a \cos\theta_a \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Finally, convert back to spherical coordinates:

$$\theta_{\odot a} = \cos^{-1} \frac{z'}{\sqrt{x'^2 + y'^2 + z'^2}}$$
$$\Phi_{\odot a} = \tan^{-1} \frac{y'}{x'}$$

$\Phi_{\odot a}$ must be between 0 and 360 and then converted back to mathematic negative coordinate system (i.e. subtract it from 360).

Source: Andre Ehrlich, Leipzig Institute for Meteorology (a.ehrlich@uni-leipzig.de)

References:

7.4 Scattering Angles

Algorithm name: scattering_angles

Category: Radiation

Summary: Calculates the scattering angle for each pixel of an image given the camera viewing angle and solar vector.

Inputs:

n_x	Coeff	Number of pixels in x dimension
n_y	Coeff	Number of pixels in y dimension
θ_c	Array[n_x, n_y]	Camera viewing zenith angle [degrees]
Φ_c	Array[n_x, n_y]	Camera viewing azimuth angle [degrees] (0°= flight direction)
$\theta_\odot$	Coeff	Solar zenith angle [degrees]
$\Phi_\odot$	Coeff	Solar azimuth angle [degrees] (0°= North)

Outputs:

θ_{scat}	Array[n_x, n_y]	Scattering angles of each pixel [degrees]

Formula:

$$\theta_{scat} = \cos^{-1}(-\sin\theta_\odot\cos\Phi_\odot\sin\theta_c\cos\Phi_c - \sin\theta_\odot\sin\Phi_\odot\sin\theta_c\sin\Phi_c + \cos\theta_\odot\cos\theta_c)$$

Source: Andre Ehrlich, Leipzig Institute for Meteorology (a.ehrlich@uni-leipzig.de)

References:

7.5 Solar Vector Calculation (Blanco)

Algorithm name: solar_vector_blanco

Category: Radiation

Summary: This algorithm comuptes the current solar vector, given current date, time, latitude and longitude. Algorithm is most accurate between 1999-2005, but calculations out to 2015 show the solar vector can be determined with an error of less than 0.5 minutes of arc.

Inputs:

Date_time	Vector	ISO String of current date/time in UTC [yyyymmddThhmmss]
lat	Vector	Latitude [degrees]
long	Vector	Longitude [degrees]

Outputs:

ra	Vector	Right ascension [radians]
δ	Vector	Declination [radians]
θ_z	Vector	Solar Zenith [radians]
γ	Vector	Solar Azimuth [radians]

Formula:

$$\begin{aligned} jd = &\frac{1461}{4}(y + 4800 + (m - 14)/12) + \frac{367}{12}(m - 2 - 12((m - 14)/12)) \\ &- \frac{3}{4}(y + 4900 + (m - 14)/12)/100 + d - 32075 - 0.5 + hour/24.0 \\ jd = &(1461(y + 4800 + (m - 14)/12))/4 + (367(m - 2 - 12((m - 14)/12)))/12 \\ &- (3((y + 4900 + (m - 14)/12)/100))/4 + d + 32075 - 0.5 + hour/24.0 \end{aligned}$$

where y is the year, m is the month, d is the day of the month and $hour$ is the current hour in decimal format, i.e. with minutes and seconds as fractions of an hour. Note that all divisions in this calculation are integer divisions except the last.

The ecliptic coordinates of the sun are computed from the Julian Day by:

$$\begin{aligned} n &= jd - 2451545.0 \\ \Omega &= 2.1429 - 0.0010394594n \\ L \text{ (mean longitude) } &= 4.8950630 + 0.017202791698n \\ g \text{ (mean anomaly) } &= 6.2400600 + 0.0172019699n \\ l \text{ (ecliptic longitude) } &= L + 0.03341607 \sin g + 0.00034894 \sin 2g - 0.0001134 - 0.0000203 \sin\Omega \\ ep \text{ (obliquity of the ecliptic) } &= 0.4090928 - 6.2140 \times 10^{-9}n + 0.0000396 \cos\Omega \end{aligned}$$

The conversion from ecliptic coordinates to celestial coordinates is computed by:

$$ra \text{ (right ascension)} = \tan^{-1}\left[\frac{\cos ep \sin l}{\cos l}\right]$$
$$\delta \text{ (declination)} = \sin^{-1}[\sin ep \sin l]$$

where ra must be between 0 and 2π.

The conversion between celestial coordinates to horizontal coordinates is then computed by the following equations:

$$gmst = 6.6974243242 + 0.0657098283n + hour$$
$$lmst = \frac{pi}{180}(15gmst + long)$$
$$\omega \text{ (hour angle)} = lmst - ra$$
$$\theta_z = \cos^{-1}[\cos lat \cos\omega \cos\delta + \sin\delta \sin lat]$$
$$\gamma = \tan^{-1}\left[\frac{-\sin\omega}{\tan\delta \cos lat - \sin lat \cos\omega}\right]$$
$$Parallax = \frac{EarthMeanRadius}{AstronomicalUnit}\sin\theta_z$$
$$\theta_z = \theta_z + Parallax$$

where: $EarthMeanRadius = 6371.01$ km and $AstronomicalUnit = 149597890$ km

Source:

References: Manuel Blanco-Muriel, et al., "Computing the Solar Vector," *Solar Energy* 70 (2001): 436-38. [2]

7.6 Solar Vector Calculation (Reda-Andreas)

Algorithm name: solar_vector_reda

Category: Radiation

Summary: This algorithm calculates the current solar vector based on time, latitude and longitude inputs. It accepts optional pressure and temperature arguments to correct for atmospheric refraction effects. The zenith and azimuth angle calculated by this algorithm have uncertainties equal to ±0.0003° in the period from the year -2000 to 6000.

Inputs:

Date_time	Vector	ISO String of current date/time in UTC [yyyymmddThhmmss]
lat	Vector	Latitude [degrees]
long	Vector	Longitude [degrees]
E	Vector	Elevation [m]
P	Vector, Optional	Local pressure [hPa]
T	Vector, Optional	Local temperature [°C]

Outputs:

θ	Vector	Solar Zenith [degrees]
Φ	Vector	Solar Azimuth [degrees]

Formula:

1. Calculate Julian and Julian Ephemeris Day, Century and Millennium:

   (a) Calculate Julian Day (JD):

   $$JD = \text{INT}(365.25(Y + 4716)) + \text{INT}(30.6001(M + 1)) + D + B - 1524.5$$

   where:

   - INT is the integer of the calculated terms (e.g. $8.7 = 8$, $8.2 = 8$, etc)
   - Y is the year
   - M is the month of the year. If $M <= 2$ then $Y = Y - 1$ and $M = M + 12$
   - D is the day of the month with decimal time (i.e. with fractions of the day being represented after the decimal point.)
   - B is equal to 0 for the Julian Calendar, and equal to $(2 - A + \text{INT}(A/4))$ for the Gregorian calendar, where $A = \text{INT}(Y/100)$

   (b) Calculate Julian Ephemeris Day (JDE):

   $$JDE = JD + \frac{\Delta T}{86400}$$

Where ΔT is the difference between the Earth rotation time and the Terrestrial Time. It can be calculated following the NASA "Polynomial expressions for *delta_T* (ΔT)" [12].

(c) Calculate Julian Century (JC) and the Julian Ephemeris Century (JCE) for the 2000 standard epoch:

$$JC = \frac{JD - 2451545}{36525}$$
$$JCE = \frac{JDE - 2451545}{36525}$$

(d) Calculate the Julian Ephemeris Millennium (JME) for the 2000 standard epoch:

$$JME = \frac{JCE}{10}$$

2. Calculate Earth heliocentric longitude, latitude and radius vector (L, B, and R):

   (a) Calculate $L0_i$ and $L0$:

   $$L0_i = A_i \cos(B_i + C_i \times JME)$$
   $$L0 = \sum_{i=0}^{n} L0_i$$

   Where the terms A_i, B_i and C_i are based on values found in table A4.2 of the algorithm literature [9].

   (b) Calculate the terms L1, L2, L3, L4 and L5 by using these same equations, but using the appropriate terms from the table.

   (c) Calculate the Earth heliocentric longitude (in radians):

   $$L = 10^{-8}(L0 + L1 \times JME + L2 \times JME^2 + L3 \times JME^3 + L4 \times JME^4 + L5 \times JME^5)$$

   (d) Convert L to degrees and limit between 0° and 360°.

   (e) Calculate the Earth heliocentric latitude B by using table A4.2 and repeating steps (a)-(c) using the appropriate values. Then convert B to degrees. Note that there are no $B2$ through $B5$.

   (f) Calculate the Earth radius vector R (in AU) in a similar manner by repeating steps (a)-(c) and using the appropriate values from table A4.2.

3. Calculate the geocentric longitude and latitude (Θ and β):

   $$\Theta = L + 180$$
   $$\beta = -B$$

   Where Θ must be limited between 0° and 360°.

4. Calculate the nutation in longitude and obliquity ($\Delta\psi$ and $\Delta\epsilon$):

(a) Calculate the mean elongation of the moon from the sun (in degrees):

$$X_0 = 297.85036 + 445267.11480JCE - 0.0019142JCE^2 + \frac{JCE^3}{189474}$$

(b) Calculate the mean anomaly of the sun (in degrees):

$$X_1 = 357.52772 + 35999.050340JCE - 0.0001603JCE^2 - \frac{JCE^3}{300000}$$

(c) Calculate the mean anomaly of the moon (in degrees):

$$X_2 = 134.96298 + 477198.867398JCE + 0.0086972JCE^2 + \frac{JCE^3}{56250}$$

(d) Calculate the moon's argument of latitude (in degrees):

$$X_3 = 93.27191 + 483202.017538JCE - 0.0036825JCE^2 + \frac{JCE^3}{327270}$$

(e) Calculate the longitude of the ascending node of the moon's mean orbit on the ecliptic, measured from the mean equinox of the date (in degrees):

$$X_4 = 125.04452 - 1934.136261JCE + 0.0020708JCE^2 + \frac{JCE^3}{450000}$$

(f) For each row in table A4.3, calculate the terms $\Delta\psi$ and $\Delta\epsilon$ (in 0.0001 of arc seconds):

$$\Delta\psi_i = (a_i + b_i JCE)\sin\left(\sum_{j=0}^{4} X_j Y_{i,j}\right)$$

$$\Delta\epsilon_i = (c_i + d_i JCE)\cos\left(\sum_{j=0}^{4} X_j Y_{i,j}\right)$$

where:

- a_i, b_i, c_i and d_i are the values listed in the ith row and columns a, b c and d in Table A4.3.
- X_j are the X values calculated above
- $Y_{i,j}$ are the values in row i and jth Y column in table A4.3.

(g) Calculate the nutation in longitude and obliquity (in degrees):

$$\Delta\psi = \frac{\sum_{i=0}^{63} \Delta\psi_i}{36000000}$$

$$\Delta\epsilon = \frac{\sum_{i=0}^{63} \Delta\epsilon_i}{36000000}$$

5. Calculate the true obliquity of the ecliptic (in degrees):

$$\begin{aligned} U &= JME/10 \\ \epsilon_0 &= 84381.448 - 4680.93U - 1.55U^2 + 1999.25U^3 - 51.38U^4 \\ &\quad - 249.67U^5 - 39.05U^6 + 7.12U^7 + 27.87U^8 + 5.79U^9 + 2.45U^{10} \\ \epsilon &= \epsilon_0/3600 + \Delta\epsilon \end{aligned}$$

6. Calculate the aberration correction (in degrees):

$$\Delta\tau = -\frac{20.4898}{3600R}$$

7. Calculate the apparent sun longitude (in degrees):

$$\lambda = \Theta + \Delta\psi + \Delta\tau$$

8. Calculate the apparent sidereal time at Greenwich at any given time (in degrees):

$$\nu_0 = 280.46061837 + 360.98564736629(JD - 2451545) + 0.000387933JC^2 - \frac{JC^3}{38710000}$$
$$\nu = \nu_0 + \Delta\psi\cos\epsilon$$

where ν_0 must be limited to the range from 0° to 360°.

9. Calculate the geocentric sun right ascension (in degrees):

$$\alpha = \frac{180}{\pi}\tan^{-1}\left(\frac{\sin\lambda\cos\epsilon - \tan\beta\sin\epsilon}{\cos\lambda}\right)$$

where, as before, α must be limited to the range from 0° to 360°.

10. Calculate the geocentric sun declination δ (in degrees):

$$\delta = \frac{180}{\pi}\sin^{-1}(\sin\beta\cos\epsilon + \cos\beta\sin\epsilon\sin\lambda)$$

11. Calculate the observer local hour angle (in degrees):

$$H = \nu + long - \alpha$$

Limit H from 0° to 360°, and note that in this algorithm H is measured westward from south.

12. Calculate the topocentric sun right ascension and declination (in degrees):

    (a) Calculate the equatorial horizontal parallax of the sun (in degrees):

$$\xi = \frac{8.794}{3600R}$$

(b) Calculate the terms u (in radians), x and y:

$$u = \tan^{-1}(0.99664719 \tan lat)$$
$$x = \cos u + \frac{E}{6378140} \cos lat$$
$$y = 0.99664719 \sin u + \frac{E}{6378140} \sin lat$$

(c) Calculate the parallax in the sun right ascension (in degrees):

$$\Delta\alpha = \frac{180}{\pi} \tan^{-1}\left(\frac{-x \sin\xi \sin H}{\cos\delta - x \sin\xi \cos H}\right)$$

(d) Calculate the topocentric sun right ascension and declination (in degrees):

$$\alpha' = \alpha + \Delta\alpha$$
$$\delta' = \tan^{-1}\left(\frac{(\sin\delta - y \sin\xi)\cos\Delta\alpha}{\cos\delta - x \sin\xi \cos H}\right)$$

13. Calculate the topocentric local hour angle (in degrees):

$$H' = H - \Delta\alpha$$

14. Calculate the topocentric zenith angle (in degrees):

(a) Calculate the topocentric elevation angle without atmospheric correction (in degrees):

$$e_0 = \frac{180}{\pi} \sin^{-1}(\sin lat \sin\delta' + \cos lat \cos\delta' \cos H')$$

(b) Calculate the atmospheric refraction correction (in degrees):

$$\Delta e = \frac{P}{1010} \frac{283}{(T+273)} \frac{1.02}{60 \tan\left(e_0 + \frac{10.3}{e_0+5.11}\right)}$$

Note that this step is skipped if temperature and pressure are not provided by the user. Also note that the argument for the tangent is computed in degrees. A conversion to radians may be needed if required by your computer or calculator.

(c) Calculate the topocentric elevation angle (in degrees):

$$e = e_0 + \Delta e$$

(d) Calculate the topocentric zenith angle (in degrees):

$$\theta = 90 - e$$

15. Calculate the topocentric azimuth angle (in degrees):

$$\Phi = \frac{180}{\pi} \tan^{-1}\left(\frac{\sin H'}{\cos H' \sin lat - \tan\delta' \cos lat}\right) + 180$$

Limit Φ from 0° to 360°. Note that Φ is measured eastward from north.

Source:

References: Reda and Andreas, “Solar Position Algorithm for Solar Radiation Applications,” National Renewable Energy Laboratory, Revised 2008, accessed February 14, 2012, http://www.nrel.gov/docs/fy08osti/34302.pdf. [9]

7.7 Blackbody Temperature

Algorithm name: temp_blackbody

Category: Radiation

Summary: Calculates the blackbody temperature for a given radiance at a specific wavelength.

Inputs:

rad	Vector	Blackbody radiance [W m-2 sr-1 nm-1]
λ	Coeff	Wavelength [nm]

Outputs:

T	Vector	Temperature [K]

Formula: After converting λ to m and *rad* to W m-3 sr-1, the blackbody temperature is calculated by:

$$T = \frac{hc}{k_B \lambda \ln(\frac{2hc^2}{\lambda^5 rad} + 1)}$$

where c is the speed of light in m/s, h is the Planck constant in J s and k_B is the Boltzmann constant in J/K.

Source: Andre Ehrlich, Leipzig Institute for Meteorology (a.ehrlich@uni-leipzig.de)

References:

Chapter 8

Quality Control

8.1 Check navigation data for inconsistencies

Algorithm name: nav_chk

Category: Quality Control

Summary: Tests navigation file (position and attitude) for inconsistencies and corrects them. The code is based on a HyMap *gps File.

Inputs: *.gps file plus the number of image lines according to the ENVI header of the related image data. The *.gps file is a multi-column ASCII file derived by HyVista Corp. proprietary software, which synchronises times and generates an output which is indexed by scan line number. The table below shows the list of parameters.

Parameters	Example	Description
Line	1	Scan line number
UTC Time	48835.0462/20/5/2004	Time of day in seconds/day/month/year
VME Time	929386852.0	Internal computer tick time in microseconds
IMU Time	2048825953.1	Internal IMU time in microseconds
Latitude	48.03321015	Decimal degrees (positive = north, negative = south)
Longitude	11.28140200	Decimal degrees (positive = east, negative = west)
Altitude	2970.79892155	Meters above MSL
Pitch	0.22235917	Decimal degrees (positive = nose up)
Roll	0.54269902	Decimal degrees (positive = right wing up)
Heading	0.37774316	Decimal degrees (positive = N-E-S direction, negative = N-W-S direction)
True Track	1.00507651	Decimal degrees (0 to 360)
Ground Speed	72.90907700	Meters / second
Sat	5	Number of satellites being received
DGPS	1	DGPS status: 1 = DGPS being received 0 = no DGPS received

Outputs: status file → template+'_status'
If applicable: corrected gps file
backup of original .gps → filename.gps_original

Formula: test & correct the following

- point or colon - separator in .gps =¿ error catched in hymap_read_gps.pro corrected when re-writing the .gps-file anyway

- #lines in image = #lines in gps
  if too many gps-lines: truncate lines at beginning (like Hyvista does)
  if too few gps-lines: adding extrapolated lines at end
- invalid start / end time: calculating average timestep & using last relieable line
- data gaps (indicated by identical time): interpolate info

Source: DLR-DFD

References: EUFAR FP7 - DJ2.2.2 - Quality Layers for VITO, DLR, INTA and PML

8.2 Additional consistency check & QA for navigation data (no correction!)

Algorithm name: nav_const

Category: Quality Control

Summary: Tests navigation file (position and attitude) for consistency. The code is based on a HyMap *gps File.
This check can be performed after nav_chk.pro.

Inputs: *.gps file. The *.gps file is a multi-column ASCII file derived by HyVista Corp. proprietary software, which synchronises times and generates an output which is indexed by scan line number. The table below shows the list of parameters.

Parameters	Example	Description
Line	1	Scan line number
UTC Time	48835.0462/20/5/2004	Time of day in seconds/day/month/year
VME Time	929386852.0	Internal computer tick time in microseconds
IMU Time	2048825953.1	Internal IMU time in microseconds
Latitude	48.03321015	Decimal degrees (positive = north, negative = south)
Longitude	11.28140200	Decimal degrees (positive = east, negative = west)
Altitude	2970.79892155	Meters above MSL
Pitch	0.22235917	Decimal degrees (positive = nose up)
Roll	0.54269902	Decimal degrees (positive = right wing up)
Heading	0.37774316	Decimal degrees (positive = N-E-S direction, negative = N-W-S direction)
True Track	1.00507651	Decimal degrees (0 to 360)
Ground Speed	72.90907700	Meters / second
Sat	5	Number of satellites being received
DGPS	1	DGPS status: 1 = DGPS being received 0 = no DGPS received

Outputs: if (KEYWORD_SET(gps_err_array)) → QC array
otime, lat, lon, alt, pit, rol, heading, track, speed, sat, dgps
Values: 0:OK 1:minor problem 2:major problem
if (KEYWORD_SET(gps_data)) → gps data as array
otime, lat, lon, alt, pit, rol, heading, track, speed, sat, dgps

Formula: test & report the following

- if data range is not plausible

- if change between steps > threshold:
  latlon, alt, pit, rol, heading, track, speed
- uncorrectable errors in:
  time, latlon, alt, pit, rol, heading, track, speed, sat, dgps

Source: DLR-DFD

References: EUFAR FP7 - DJ2.2.2 - Quality Layers for VITO, DLR, INTA and PML

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